Exercise 33

If $p(x)$ is the total value of the production when there are x workers in a plant, then the *average productivity* of the workforce at the plant is

$$A(x) = \frac{p(x)}{x}$$

(a) Find $A'(x)$. Why does the company want to hire more workers if $A'(x) > 0$?

(b) Show that $A'(x) > 0$ if $p'(x)$ is greater than the average productivity.

Solution

Part (a)

Take the derivative of $A(x)$ to get $A'(x)$.

$$\begin{aligned} A'(x) &= \frac{d}{dx}\left[\frac{p(x)}{x}\right] \\ &= \frac{\left[\dfrac{d}{dx}p(x)\right]x - \left[\dfrac{d}{dx}(x)\right]p(x)}{x^2} \\ &= \frac{p'(x)x - (1)p(x)}{x^2} \\ &= \frac{p'(x)x - p(x)}{x^2} \end{aligned}$$

$A'(x)$ is the rate that the average productivity increases as the number of employees increases. If $A'(x) > 0$, then the company will produce more value on average by hiring additional employees.

Part (b)

Set $A'(x) > 0$ and find the condition for which this inequality is true. Note x and x^2 are positive.

$$\begin{aligned} A'(x) &> 0 \\ \frac{p'(x)x - p(x)}{x^2} &> 0 \\ p'(x)x - p(x) &> 0 \\ p'(x)x &> p(x) \\ p'(x) &> \frac{p(x)}{x} \\ p'(x) &> A(x) \end{aligned}$$

Therefore, $A'(x) > 0$ if $p'(x)$ is greater than the average productivity.